UNIVERSITÄT KLAGENFURT

# Solving problems by searching

Chapter 3

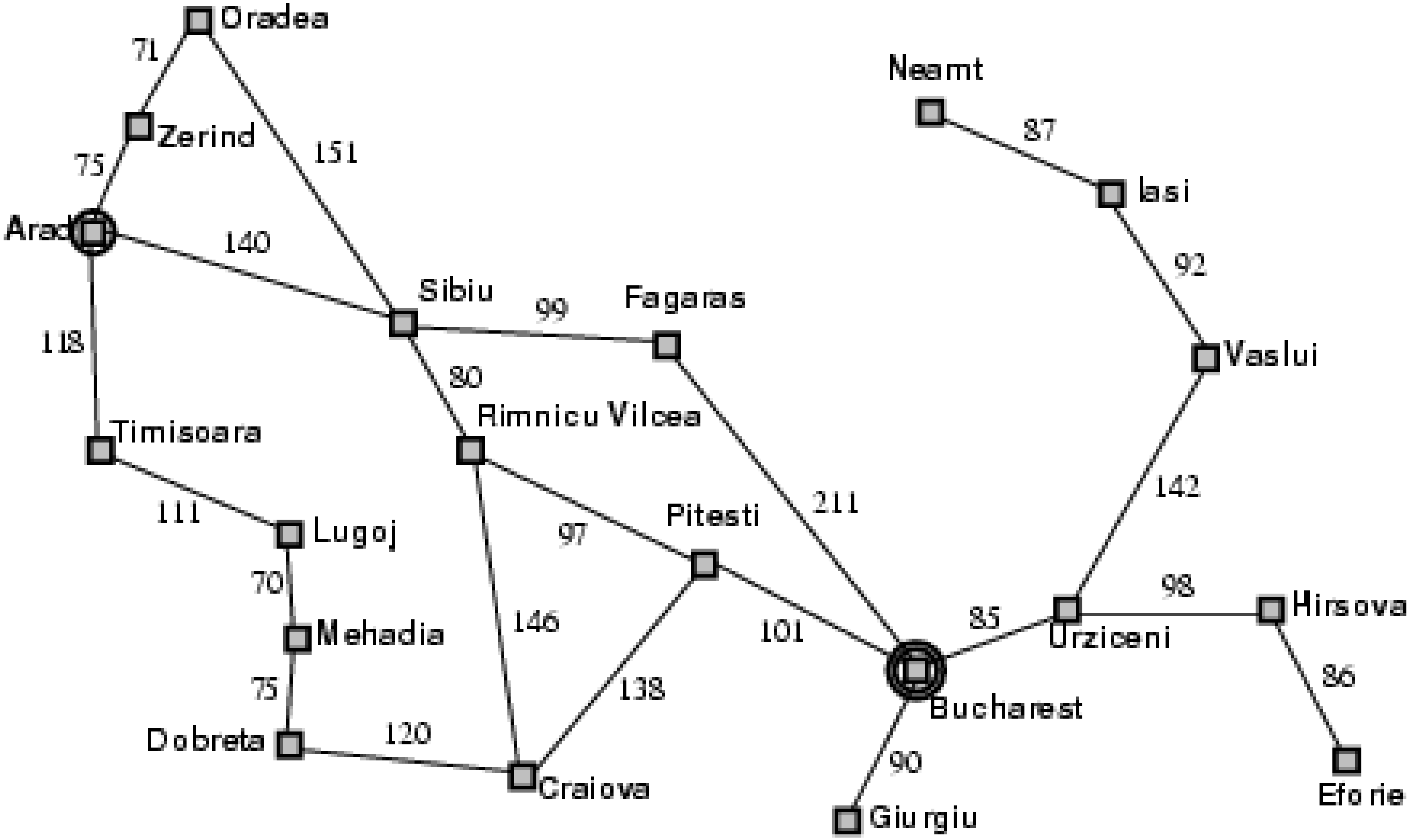
# Outline

- Problem-solving agents
- Problem types
- Problem formulation
- Example problems
- Basic search algorithms

# Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest
- Formulate goal:
  - be in Bucharest
- Formulate problem:
  - states: various cities
  - actions: drive between cities
- Find solution:
  - sequence of cities, e.g. Arad, Sibiu, Fagaras, Bucharest

# Example: Romania



# Problem-solving agent

Restricted form of general agent; solution executed “eyes closed“:

**function** SIMPLE-PROBLEM-SOLVING-AGENT(*percept*) **return** an action

**static**: *seq*, an action sequence

*state*, some description of the current world state

*goal*, a goal

*problem*, a problem formulation

*state* ← UPDATE-STATE(*state*, *percept*)

**if** *seq* is empty **then**

*goal* ← FORMULATE-GOAL(*state*)

*problem* ← FORMULATE-PROBLEM(*state*,*goal*)

*seq* ← SEARCH(*problem*)

*action* ← FIRST(*seq*)

*seq* ← REST(*seq*)

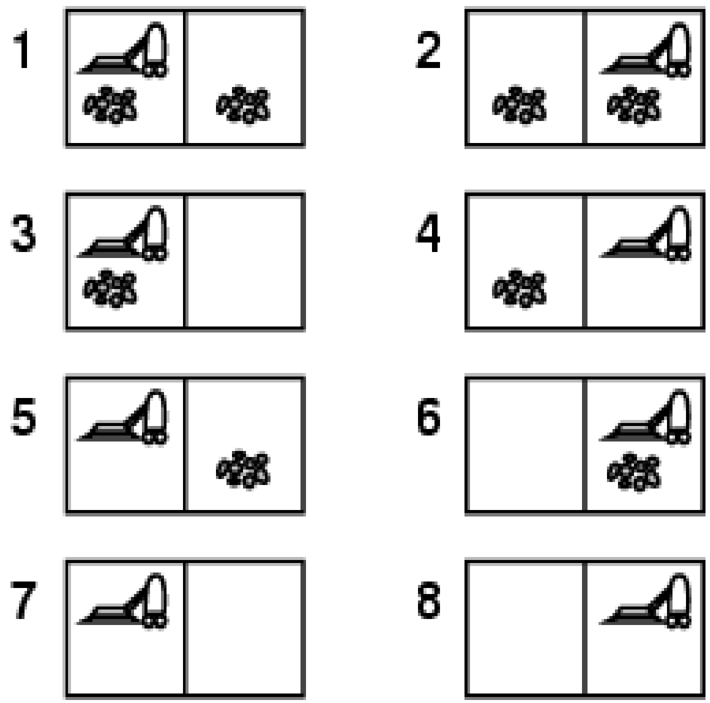
return *action*

# Problem types

- Deterministic, fully observable → single-state problem
  - Agent knows exactly which state it will be in; solution is a sequence
- Non-observable → sensor-less problem (conformant problem)
  - Agent may have no idea where it is; solution is a sequence
- Partially observable → contingency problem
  - Perception provides new information about current state
  - Often interleave search, execution
- Unknown state space → exploration problem
  - When states and actions of the environment are unknown

# Example: vacuum world

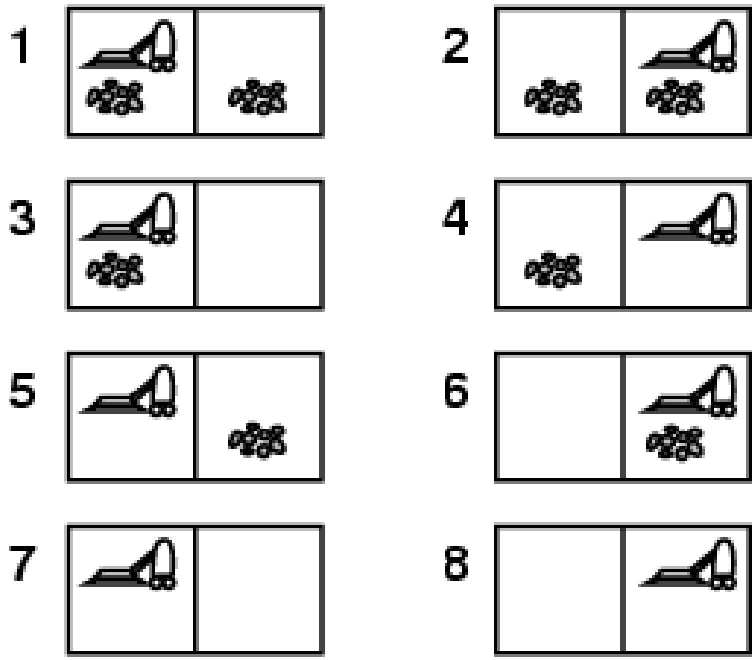
- Single-state, start in #5.
  Solution?



# Example: vacuum world

- Single-state, start in #5.
  Solution? *[Right, Suck]*

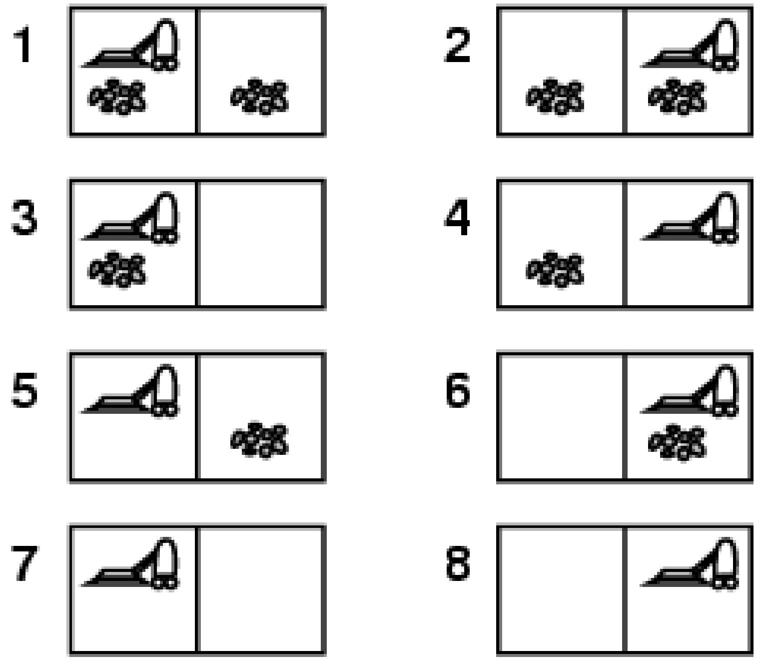
- Sensorless, start in {*1,2,3,4,5,6,7,8*} e.g.,
  *Right* goes to {*2,4,6,8*}
  Solution?



# Example: vacuum world

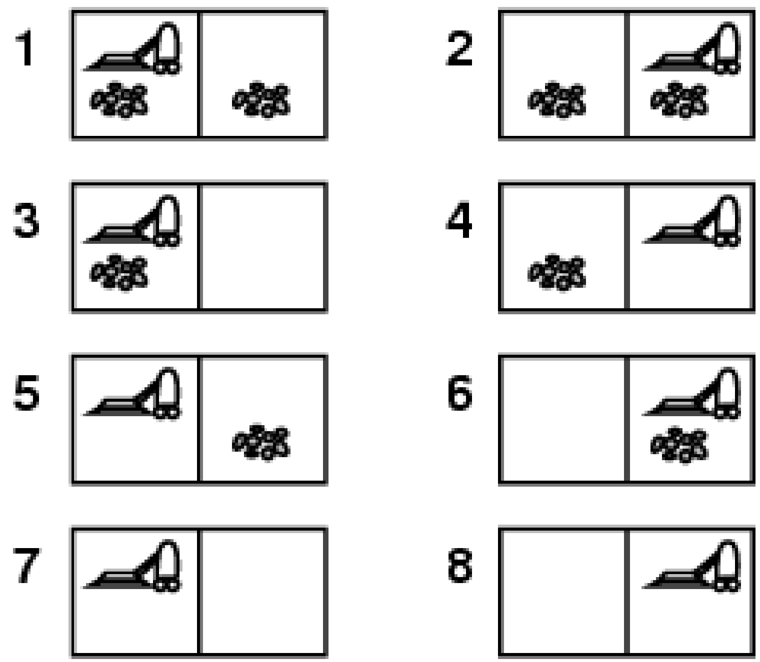
- Sensorless, start in {*1,2,3,4,5,6,7,8*} e.g., *Right* goes to {*2,4,6,8*}
  Solution?
  *[Right,Suck,Left,Suck]*

- Contingency
  - Nondeterministic: *Suck* may dirty a clean carpet
  - Partially observable: location, dirt at current location
  - Percept: *[L, Clean],* i.e., start in #5 or #7
    Solution?



# Example: vacuum world

- Sensorless, start in {*1,2,3,4,5,6,7,8*} e.g., *Right* goes to {*2,4,6,8*}
  Solution?
  *[Right,Suck,Left,Suck]*




- Contingency
  - Nondeterministic: *Suck* may dirty a clean carpet
  - Partially observable: location, dirt at current location.
  - Percept: *[L, Clean],* i.e., start in #5 or #7
    Solution? *[Right, **if** dirt **then** Suck]*

# Single-state problem formulation

A problem is defined by four items:

1. initial state, e.g. "at Arad"
2. actions or successor function *S(x)* = set of action–state pairs
   - e.g., *S(Arad)* = {*<Arad → Zerind, Zerind>, …* }
3. goal test, can be
   - explicit, e.g., *x* = "at Bucharest"
   - implicit, e.g., *Checkmate(x)*
4. path cost (additive)
   - e.g., sum of distances, number of actions executed, etc.
   - *c(x,a,y)* is the step cost, assumed to be ≥ 0

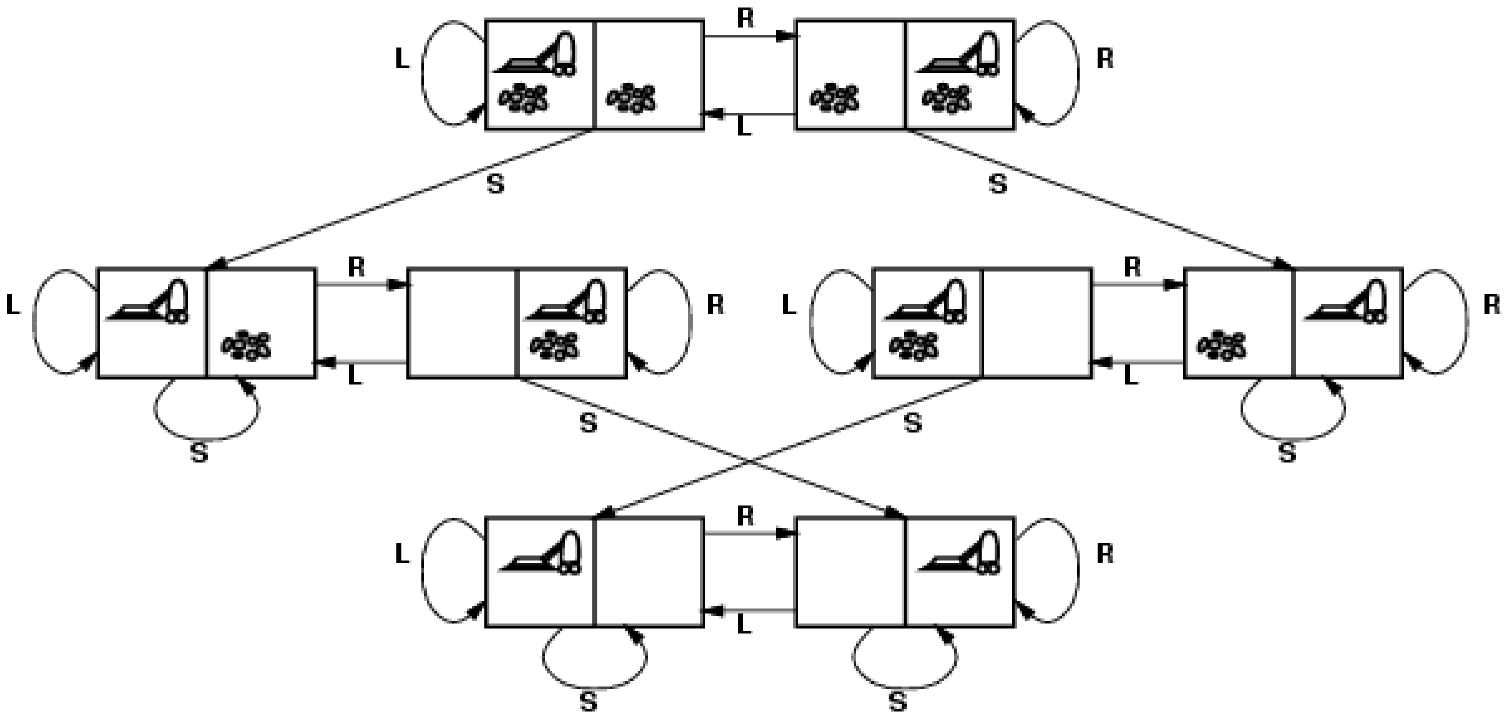
- A solution is a sequence of actions leading from the initial state to a goal state

# Selecting a state space

- Real world is absurdly complex

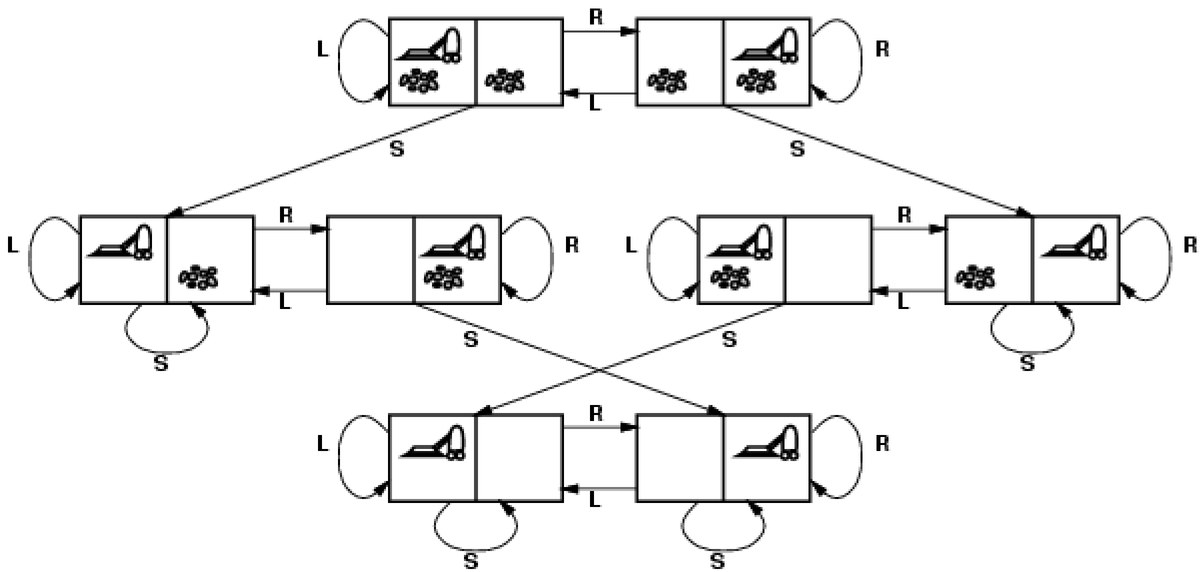
  → State space must be abstracted for problem solving
- (Abstract) state corresponds to set of real states
- (Abstract) action corr. to complex combination of real actions
  - E.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc.
- For guaranteed realizability, any real state "in Arad“ must get to some real state "in Zerind"
- (Abstract) solution corresponds to
  - Set of real paths that are solutions in the real world
- Each abstract action should be "easier" than the original problem

# Vacuum world state space graph



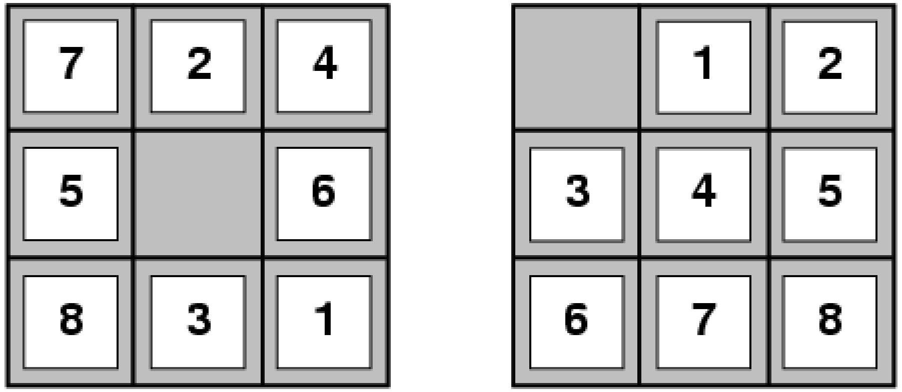

- States?
- Actions?
- Goal test?
- Path cost?

# Vacuum world state space graph




- States? two locations, dirt, and robot location
- Actions? *Left*, *Right*, *Suck*
- Goal test? no dirt at all locations
- Path cost? 1 per action

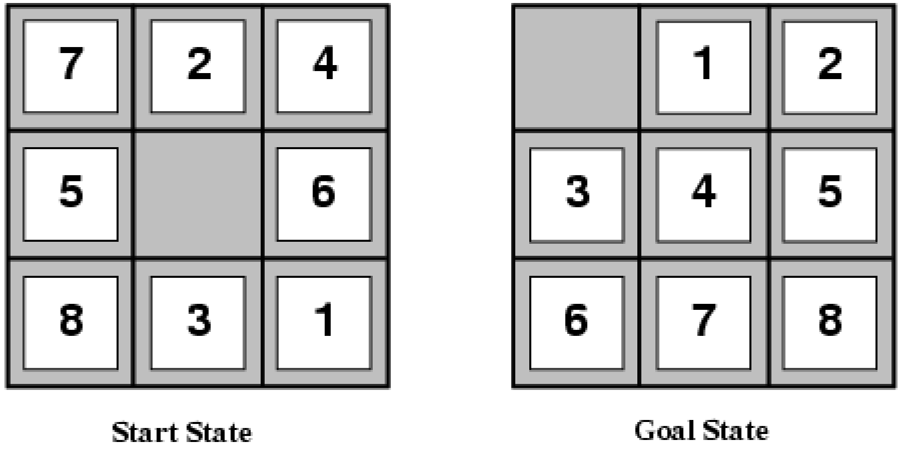
# Example: The 8-puzzle



Start State Goal State

- States?
- Actions?
- Goal test?
- Path cost?

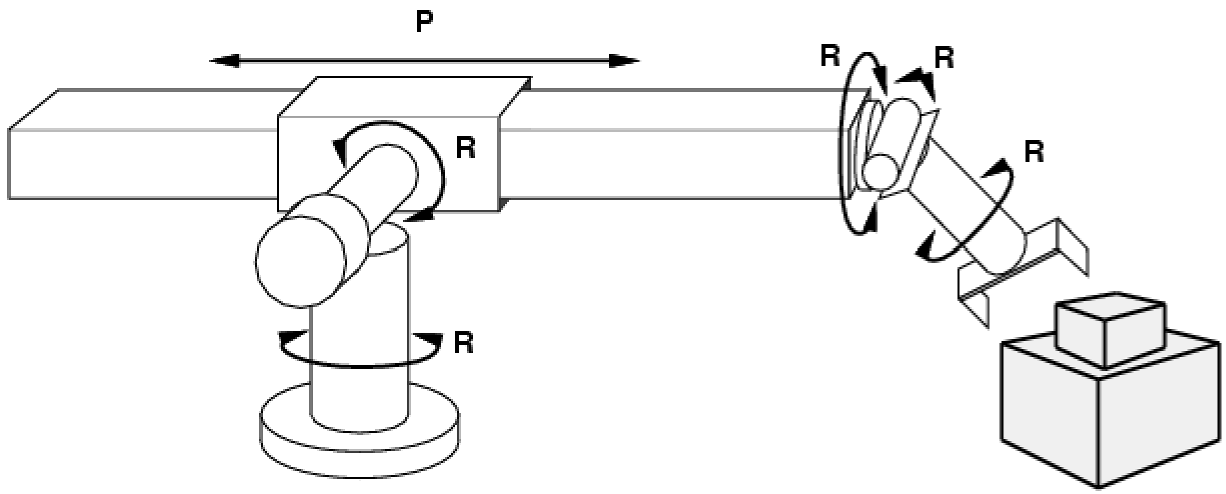
# Example: The 8-puzzle




- States? locations of tiles
- Actions? move blank left, right, up, down
- Goal test? = goal state (given)
- Path cost? 1 per move

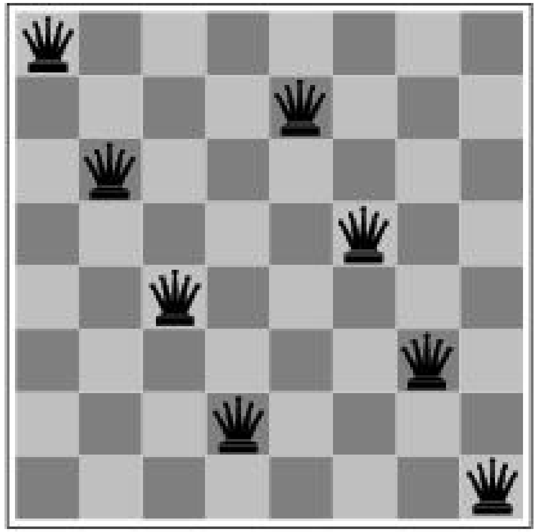
[Note: optimal solution of *n*-Puzzle family is NP-hard]

# Example: robotic assembly



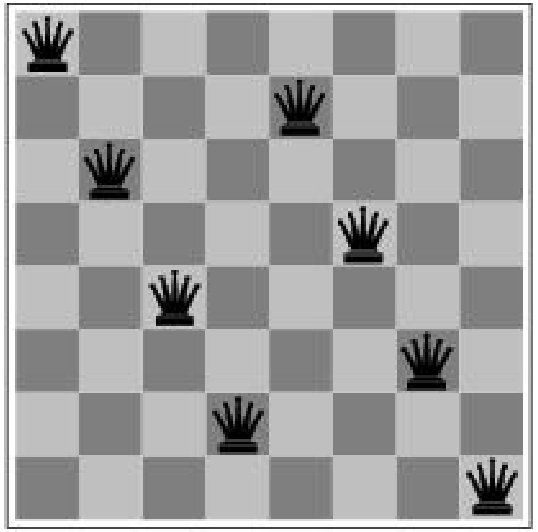

- States? real-valued coordinates of robot joint angles and parts of the object to be assembled
- Actions? continuous motions of robot joints
- Goal test? complete assembly
- Path cost? time to execute

# Example: 8-queens problem



- States?
- Actions?
- Goal test?
- Path cost?

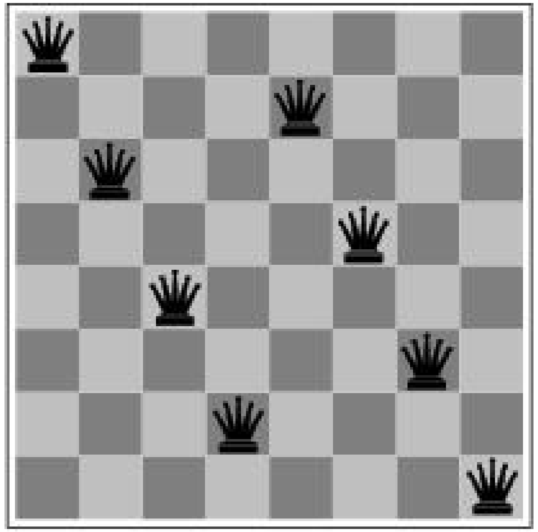
# Example: 8-queens problem



Incremental formulation vs. complete-state formulation

- States?
- Actions?
- Goal test?
- Path cost?

# Example: 8-queens problem

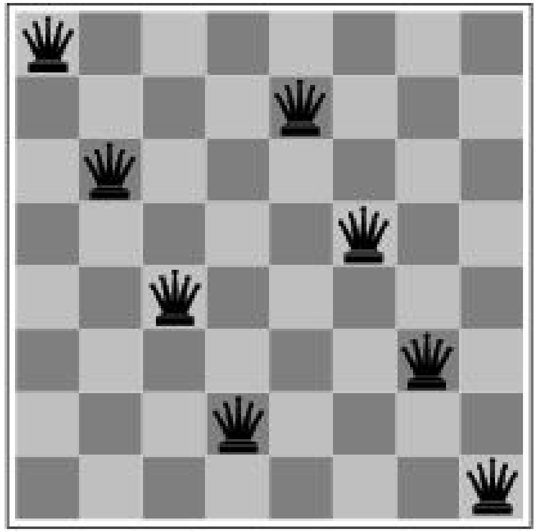


Incremental formulation

- States? any arrangement of 0 to 8 queens on the board
- Initial state? no queens
- Actions? add queen in empty square
- Goal test? 8 queens on board and none attacked
- Path cost? none

64*63*…*57 approx. 1.8 x $10^{14}$ possible sequences to investigate

# Example: 8-queens problem



Incremental formulation (alternative)

- States? *n* ($0 \leq n \leq 8$) queens on the board, one per column in the *n* leftmost columns with no queen attacking another.
- Actions? Add queen in leftmost empty column such that is not attacking other queens

# Basic search algorithms

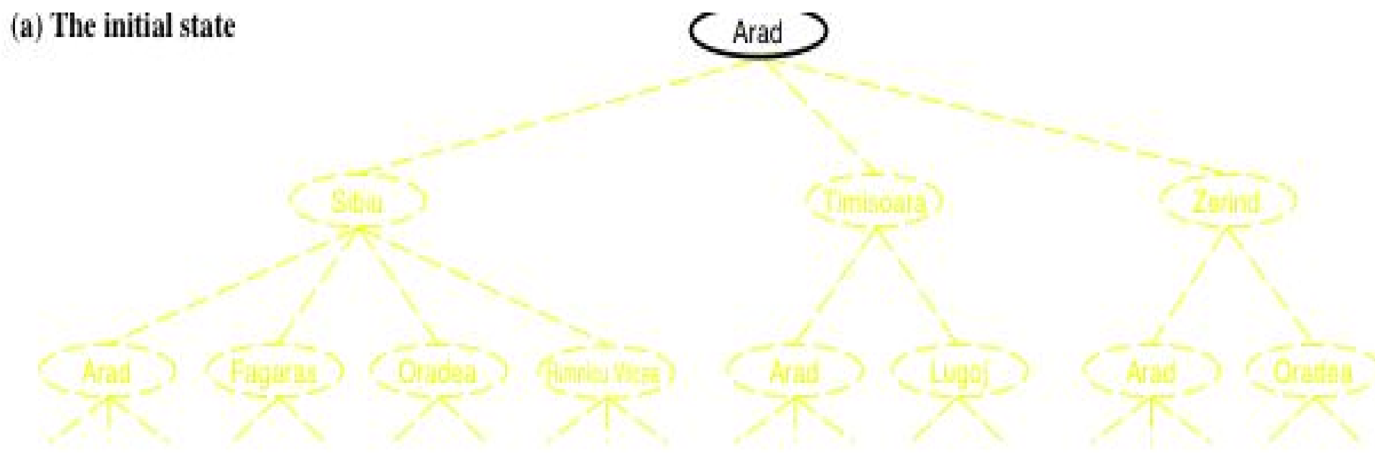
How do we find the solutions of previous problems?

- Search the state space (remember complexity of space depends on state representation)

- Here: search through *explicit tree generation*
  - ROOT= initial state.
  - Nodes and leafs generated through successor function.

- In general search generates a graph (same state through multiple paths)

# Simple tree search example




**function** TREE-SEARCH(*problem, strategy*) **return** a solution or failure

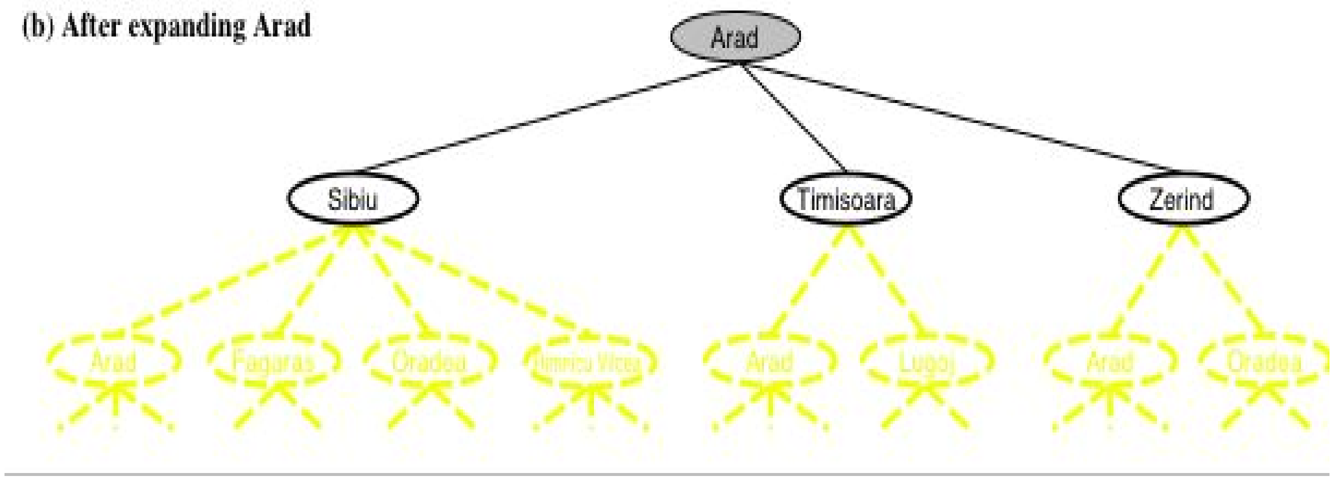
Initialize search tree to the *initial state* of the *problem*

**do**

**if** no candidates for expansion **then return** *failure*
choose leaf node for expansion according to *strategy*
**if** node contains goal state **then return** *solution*
**else** expand the node and add resulting nodes to the search tree

**enddo**

# Simple tree search example




**function** TREE-SEARCH(*problem, strategy*) **return** a solution or failure

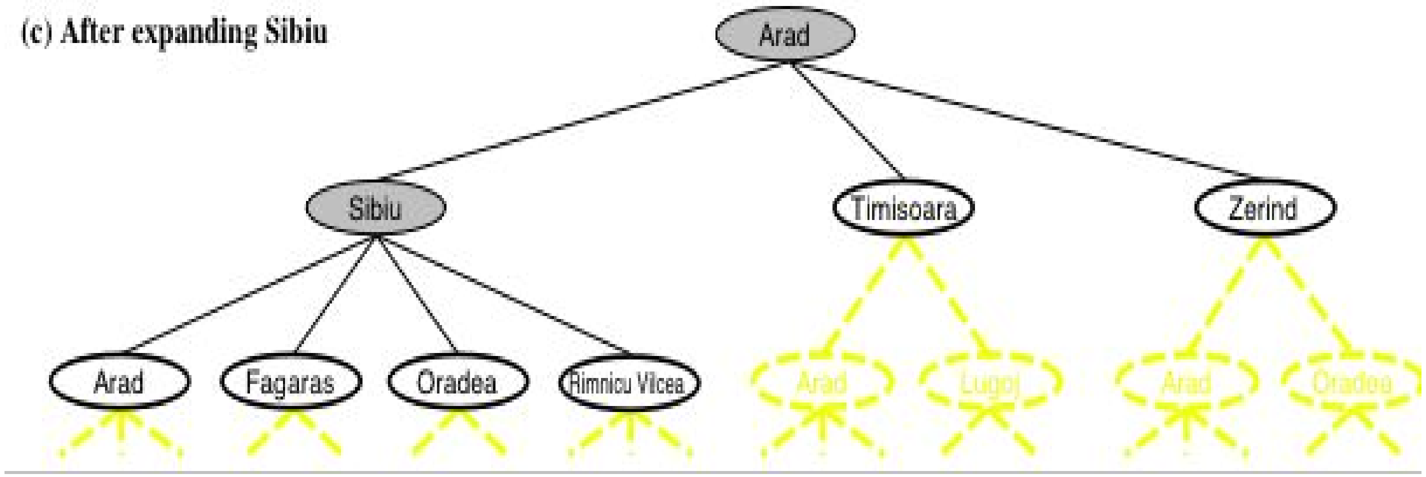
Initialize search tree to the *initial state* of the *problem*

**do**

**if** no candidates for expansion **then return** *failure*
choose leaf node for expansion according to *strategy*
**if** node contains goal state **then return** *solution*
**else** expand the node and add resulting nodes to the search tree

**enddo**

# Simple tree search example




**function** TREE-SEARCH(*problem, strategy*) **return** a solution or failure

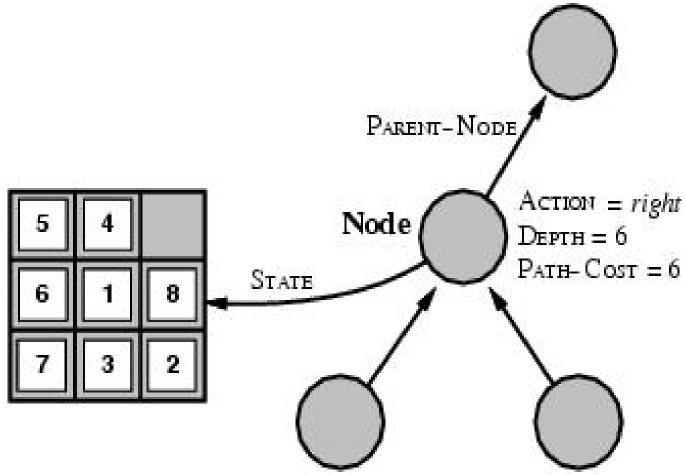
Initialize search tree to the *initial state* of the *problem*

**do**

**if** no candidates for expansion **then return** *failure*
choose leaf node for expansion according to *strategy* ←**Determines search process!!**
**if** node contains goal state **then return** *solution*
**else** expand the node and add resulting nodes to the search tree

**enddo**

# State space vs. search tree




A *state* is a (representation of) a physical configuration

A *node* is a data structure belong to a search tree

- A node has a parent, children, … and includes path cost, depth, …
- Here *node*= *<state, parent-node, action, path-cost, depth>*
- *FRINGE*= contains generated nodes which are not yet expanded
  - White nodes with black outline

# Tree search algorithm

**function** TREE-SEARCH(*problem,fringe*) **return** a solution or failure

    *fringe* ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)

    **loop do**

        **if** EMPTY?(*fringe*) **then return** failure

        *node* ← REMOVE-FIRST(*fringe*)

        **if** GOAL-TEST[*problem*] applied to STATE[*node*] succeeds

            **then return** SOLUTION(*node*)

        *fringe* ← INSERT-ALL(EXPAND(*node*, *problem*), *fringe*)

# Tree search algorithm (2)

**function** EXPAND(*node,problem*) **return** a set of nodes

*successors* $\leftarrow$ the empty set

**for each <***action***,** *result***> in** SUCCESSOR-FN**[***problem***]**(STATE[*node*]) **do**

*s* $\leftarrow$ a new NODE

STATE[*s*] $\leftarrow$ *result*

PARENT-NODE[*s*] $\leftarrow$ *node*

ACTION[*s*] $\leftarrow$ *action*

PATH-COST[*s*] $\leftarrow$ PATH-COST[*node*] + STEP-COST(*node, action,s*)

DEPTH[*s*] $\leftarrow$ DEPTH[*node*]+1

add *s* to *successors*

**return** *successors*

# Search strategies

- A search strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - *b:* maximum branching factor of the search tree
  - *d:* depth of the least-cost solution
  - *m*: maximum depth of the state space (may be ∞)

# Uninformed search strategies

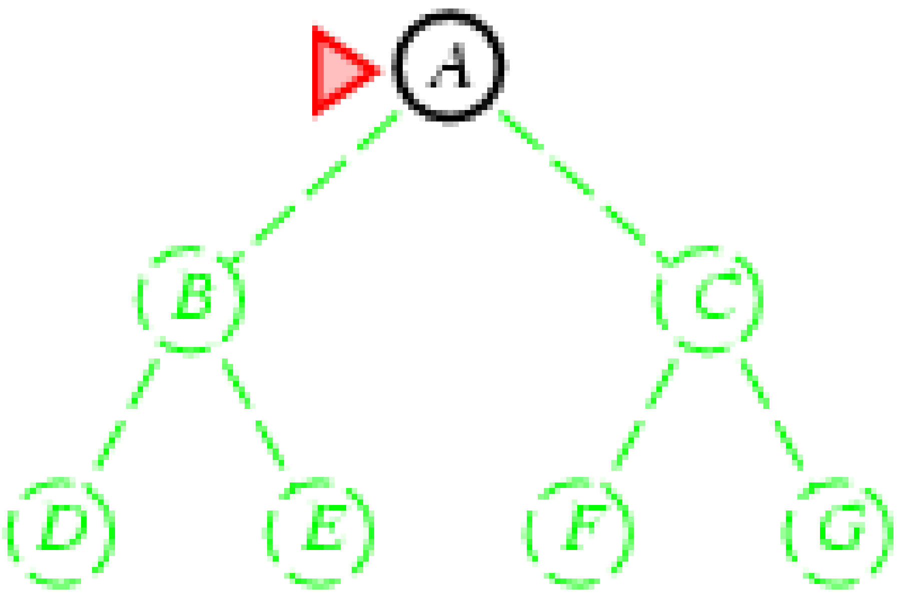
Uninformed search strategies use only the information available in the problem definition

When strategies can determine whether one non-goal state is better than another $\rightarrow$ informed search

- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search
- Bidirectional search

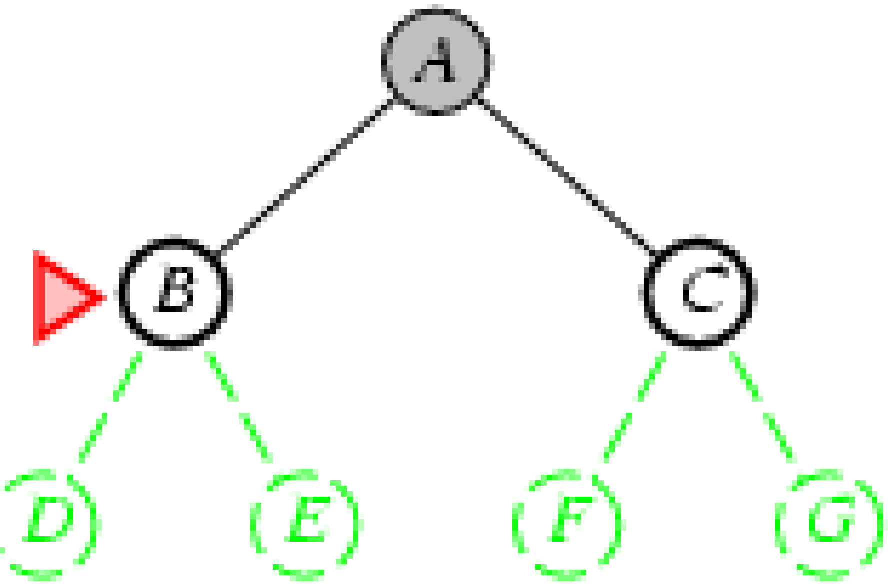
# Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end



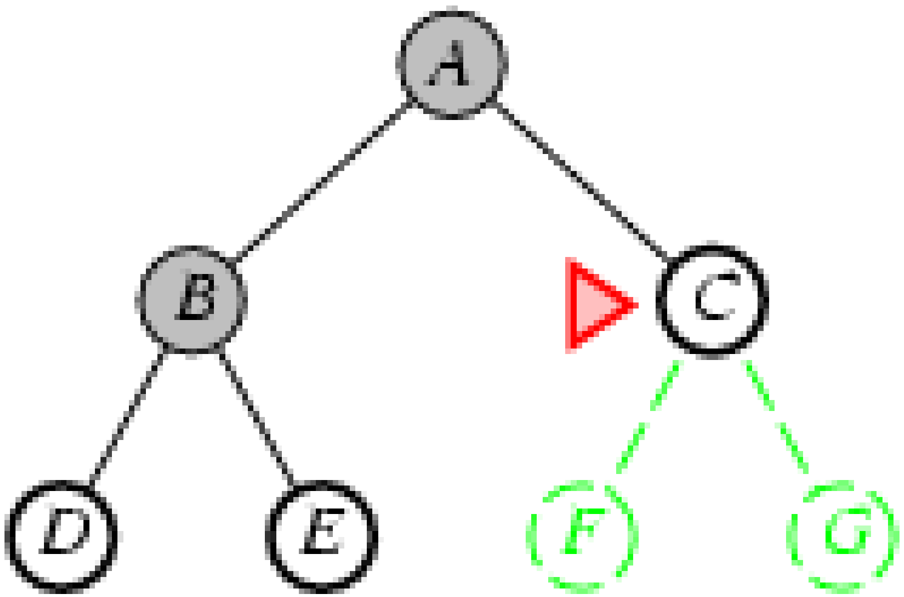
# Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end



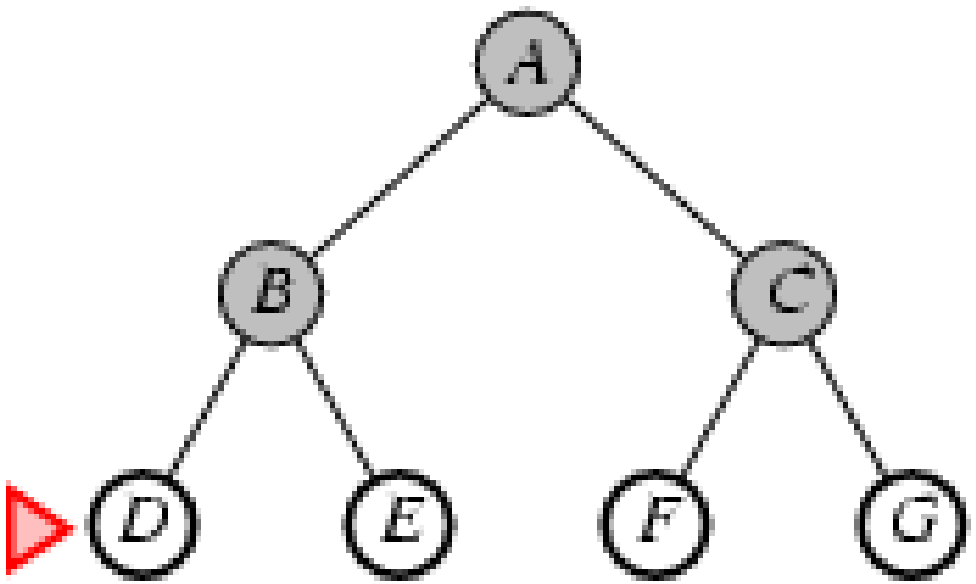
# Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end



# Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end



# Properties of breadth-first search

- Complete? Yes (if *b* is finite)
- Time? $1+b+b^2+b^3+\ldots +b^d + b(b^d-1) = O(b^{d+1})$
- Space? $O(b^{d+1})$ (keeps every node in memory)
- Optimal? Yes (if cost = 1 per step)
- Space is the bigger problem (more than time)

# BF-search; evaluation

b=10; 10.000 nodes/sec; 1000 bytes/node

| DEPTH | NODES | TIME | MEMORY |
|---|---|---|---|
| 2 | 1100 | 0.11 seconds | 1 megabyte |
| 4 | 111100 | 11 seconds | 106 megabytes |
| 6 | $10^7$ | 19 minutes | 10 gigabytes |
| 8 | $10^9$ | 31 hours | 1 terabyte |
| 10 | $10^{11}$ | 129 days | 101 terabytes |
| 12 | $10^{13}$ | 35 years | 10 petabytes |
| 14 | $10^{15}$ | 3523 years | 1 exabyte |

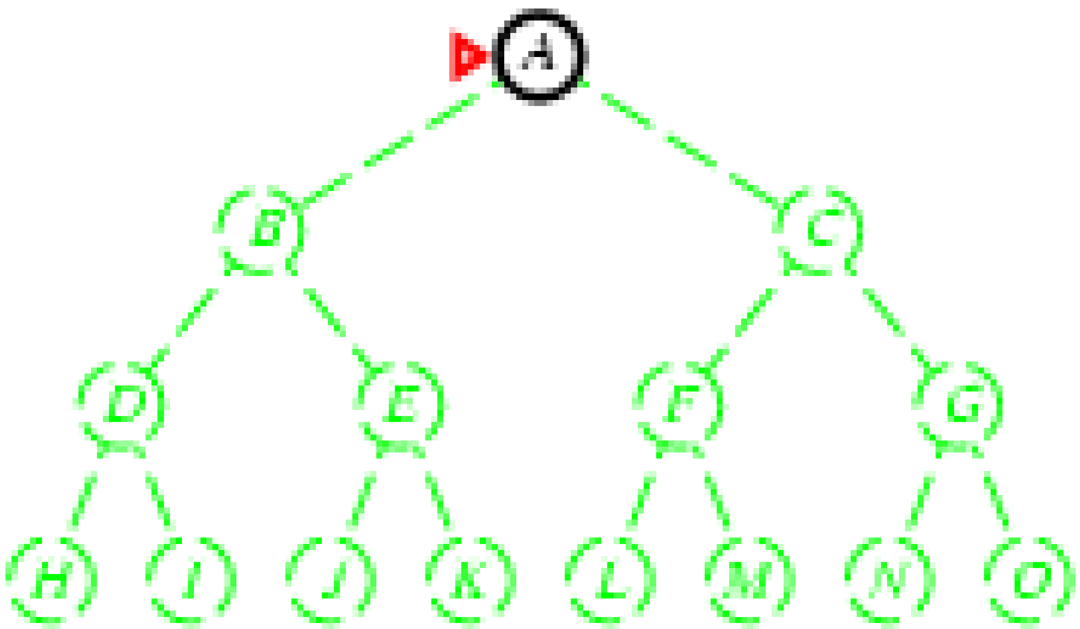
- Two lessons:
  - Memory requirements are a bigger problem than its execution time
  - Uniformed search only applicable for small instances
    -> Exploit knowledge about the problem

# Uniform-cost search

- Expand least-cost unexpanded node
- Implementation:
  - *fringe* = queue ordered by path cost
- Equivalent to breadth-first if step costs all equal
- Complete? Yes, if step cost ≥ ε
- Time? # of nodes with $g$ ≤ cost of optimal solution, $O(b^{1+floor(C^*/\varepsilon)})$ where $C^*$ is the cost of the optimal solution
- Space? # of nodes with $g$ ≤ cost of optimal solution, $O(b^{1+floor(C^*/\varepsilon)})$
- Optimal? Yes – nodes expanded in increasing order of *path costs*

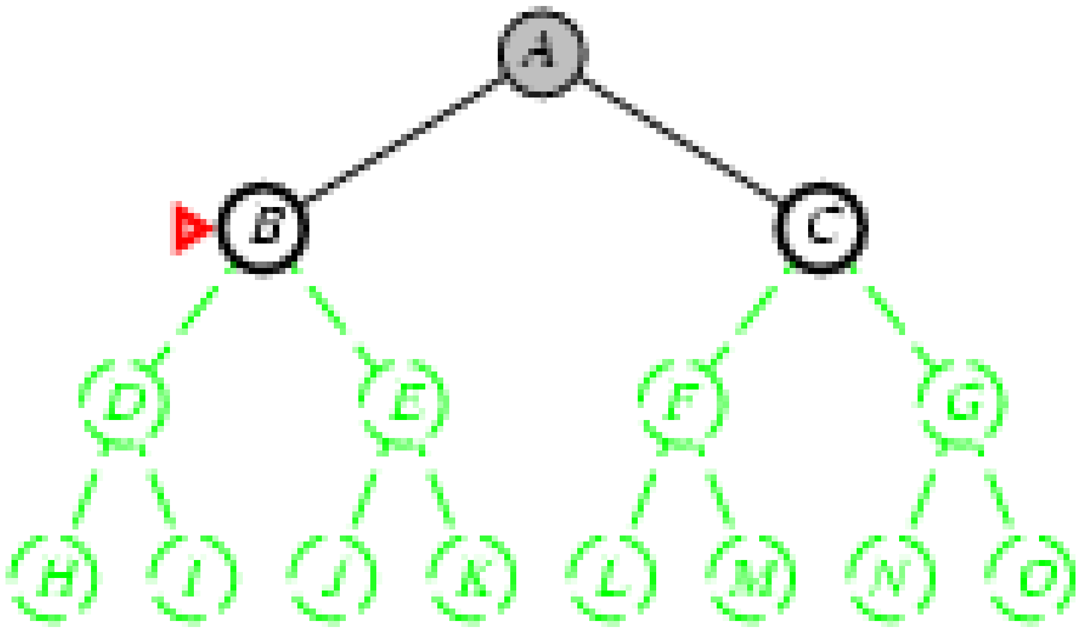
# Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



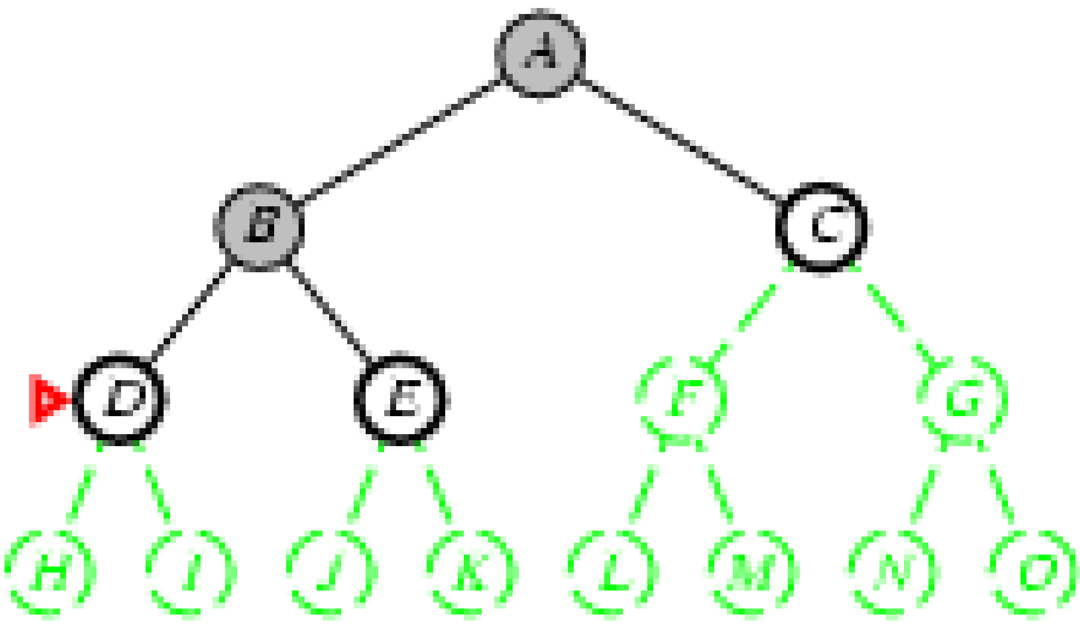
# Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



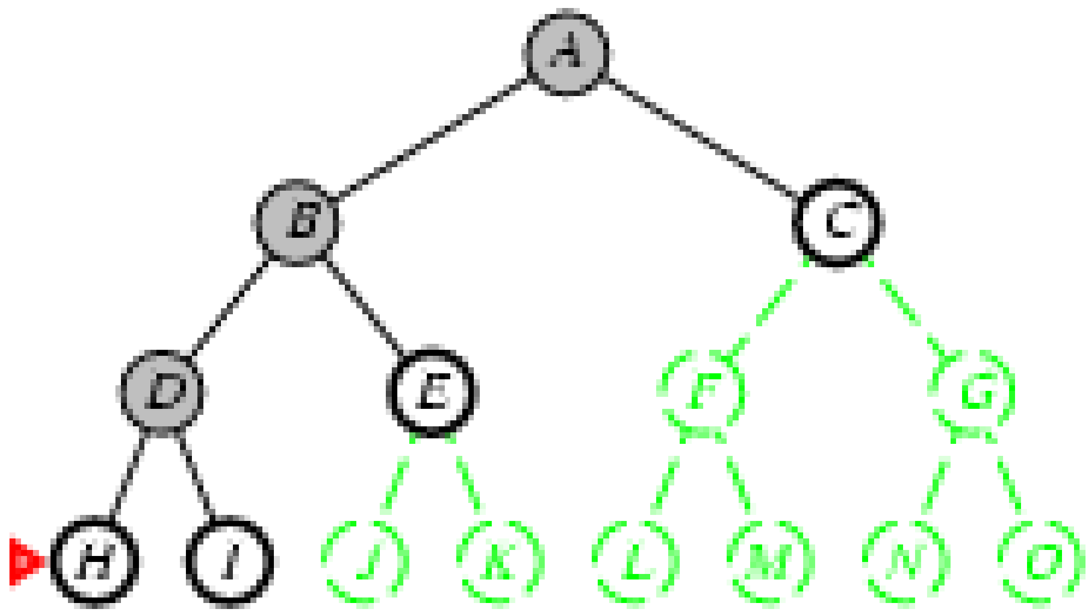
# Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



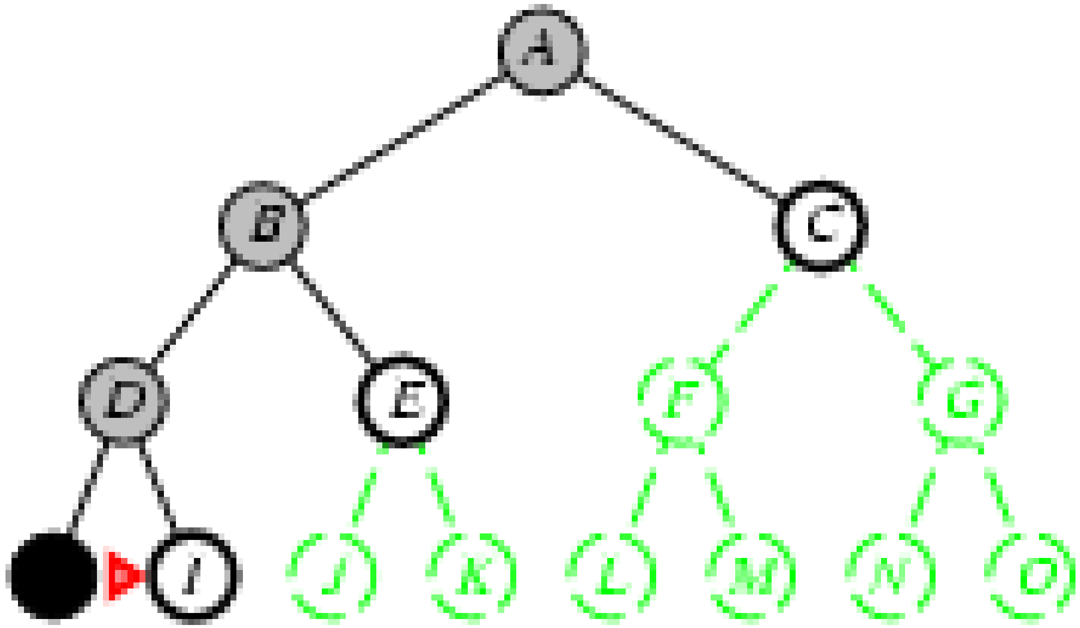
# Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



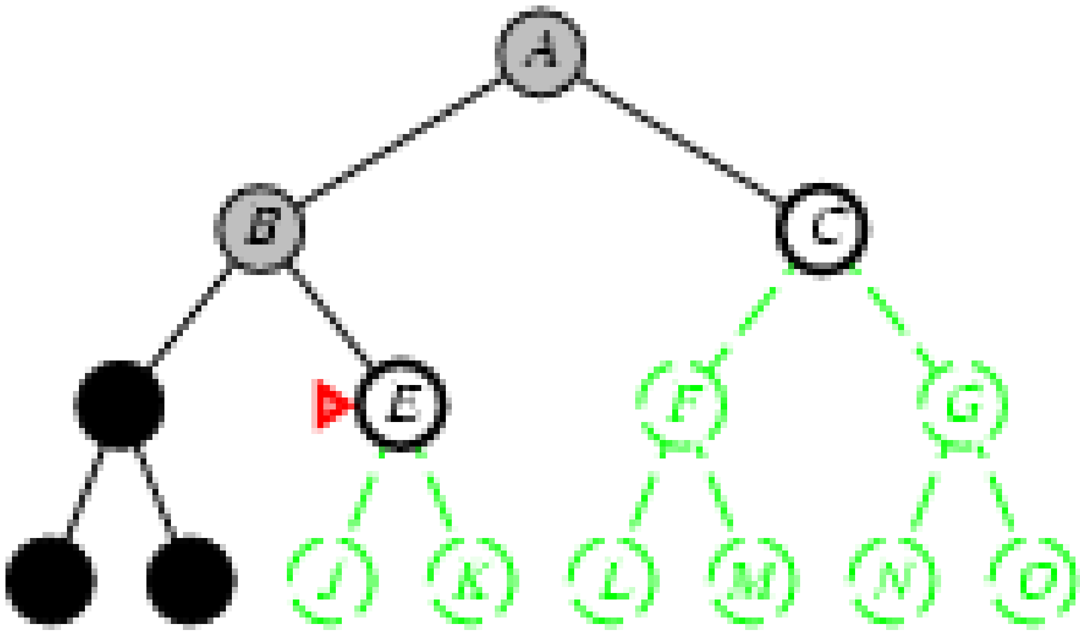
# Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



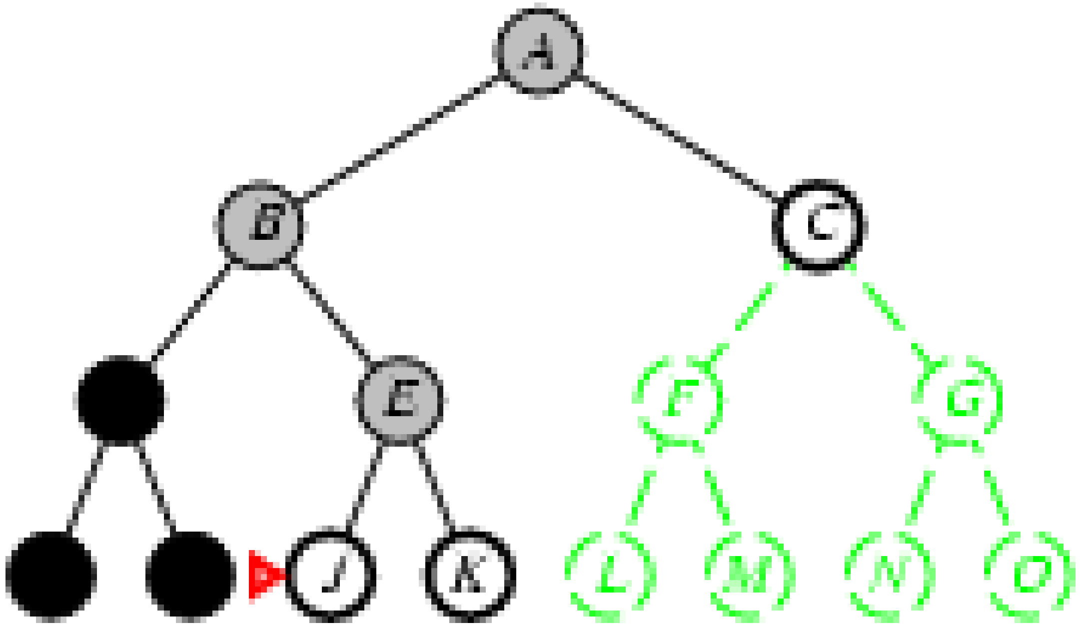
# Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



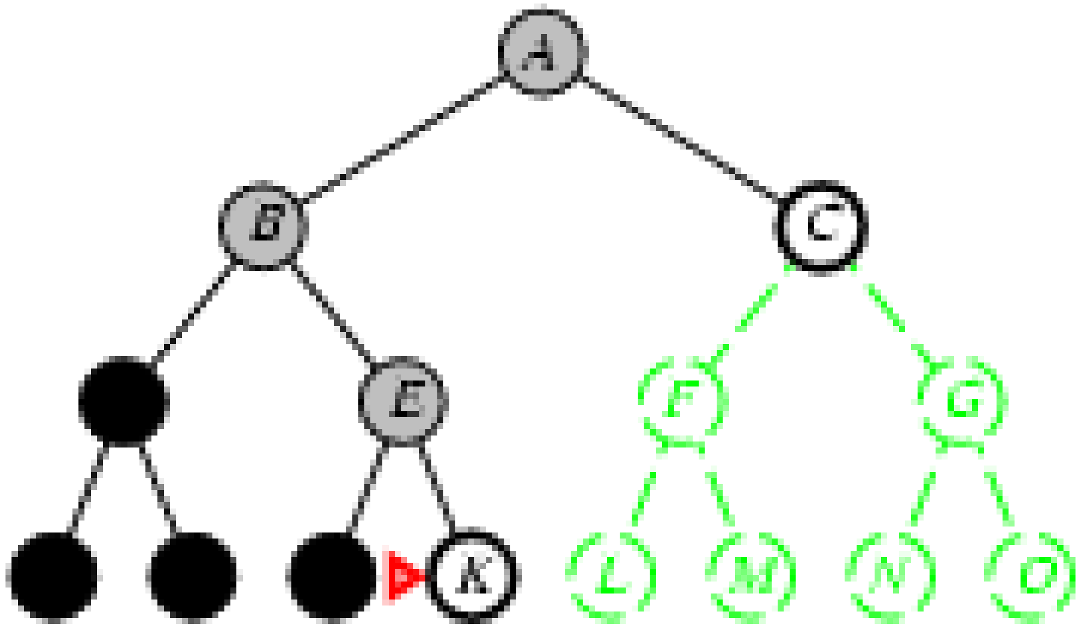
# Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



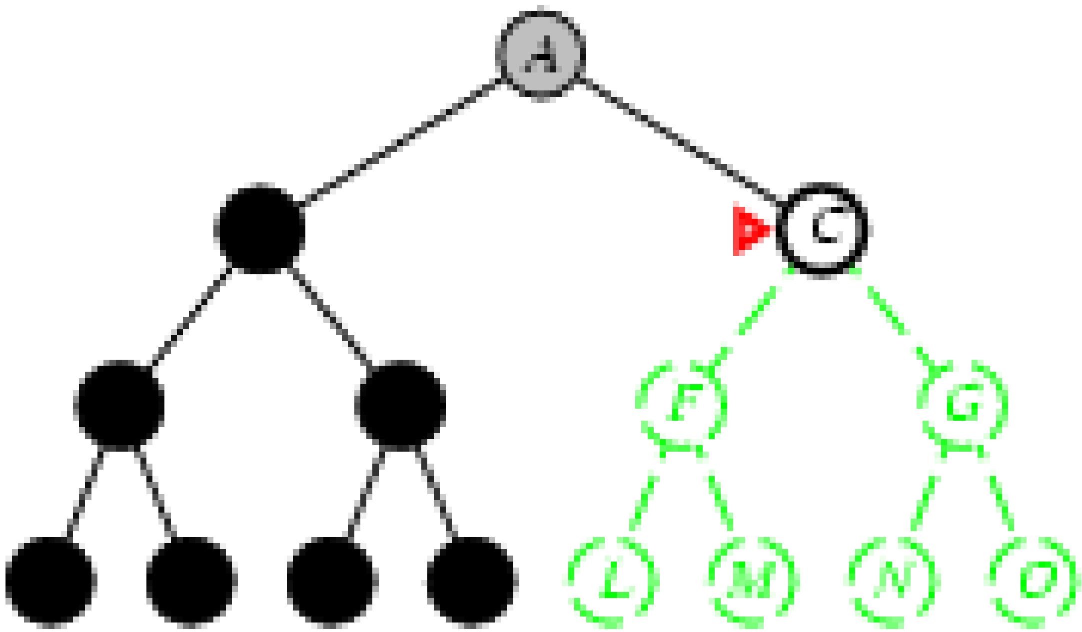
# Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



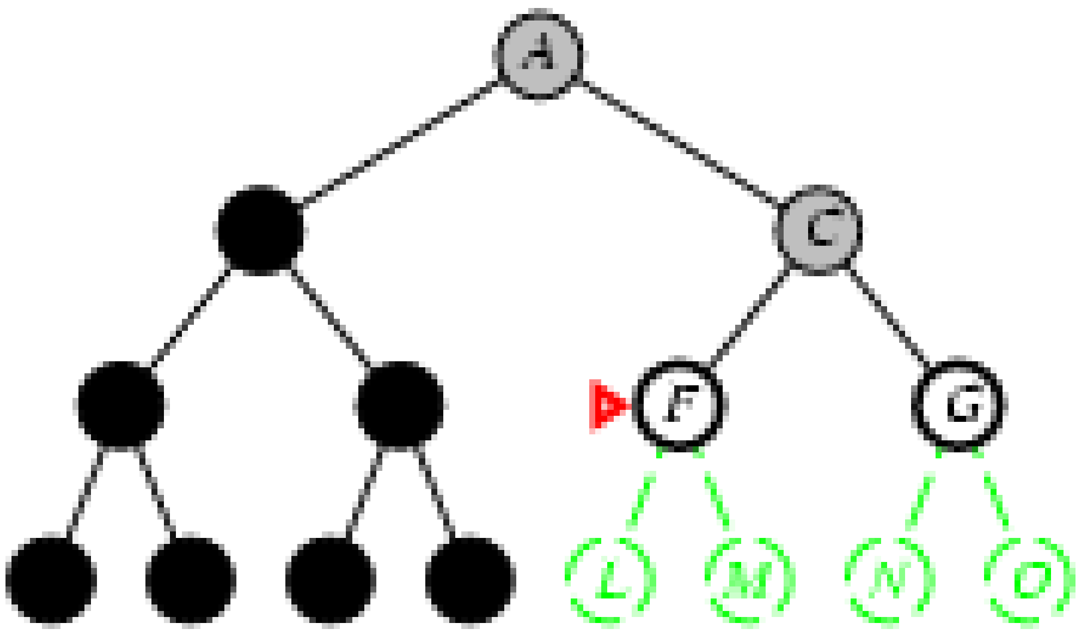
# Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



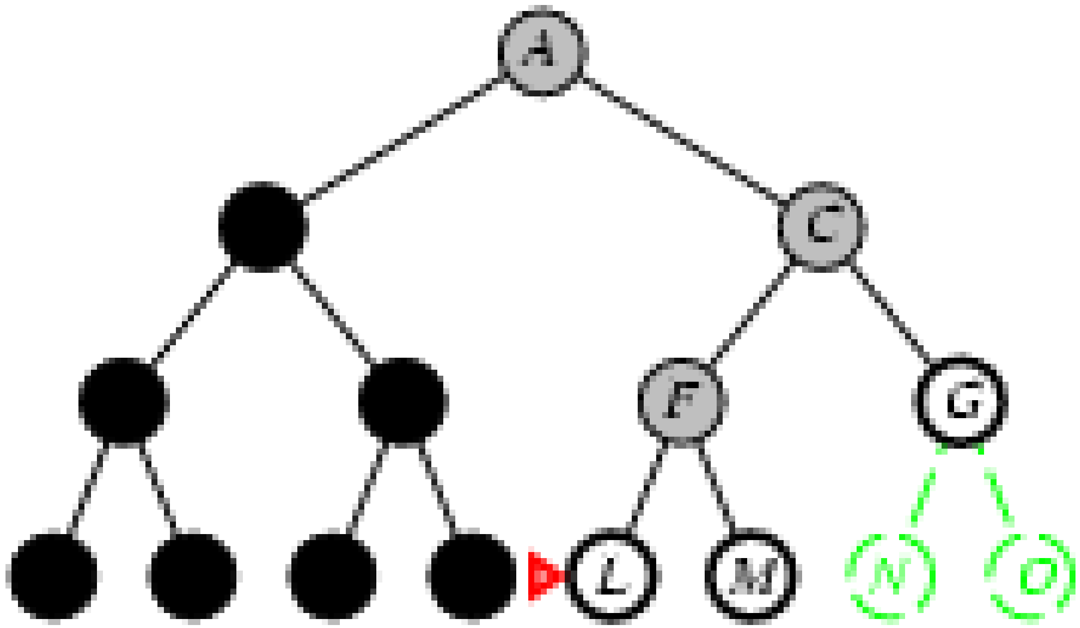
# Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



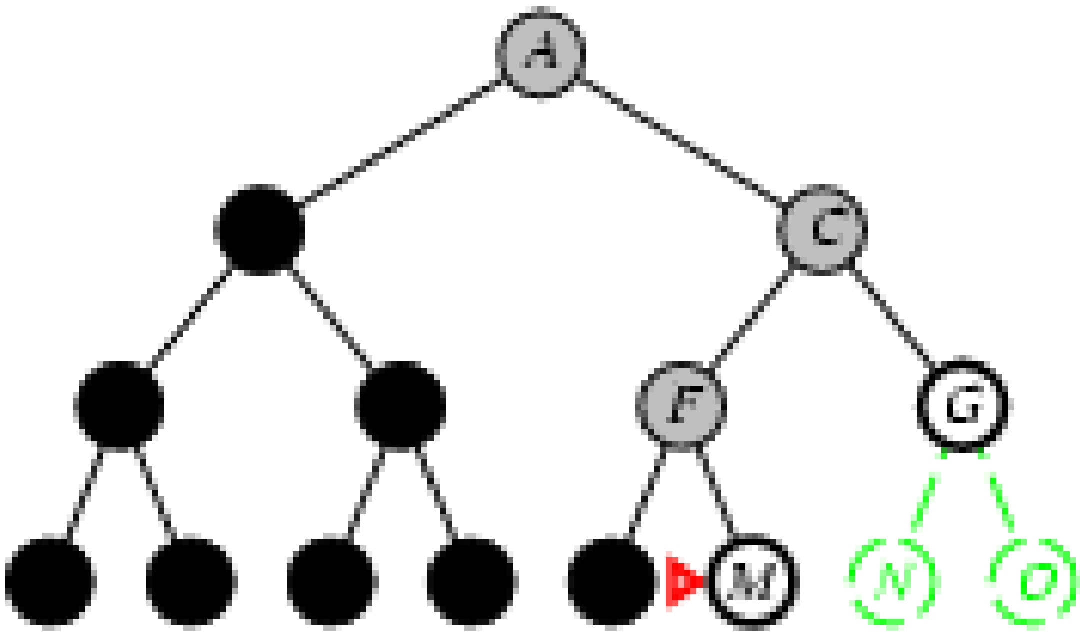
# Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



# Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



# Properties of depth-first search

- Complete? No: fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path

    → complete in finite spaces
- Time? $O(b^m)$: terrible if $m$ is much larger than $d$

  *(remember: m … maximum depth of search space)*
  - but if solutions are dense, may be much faster than breadth-first
- Space? $O(bm)$, i.e., linear space!
- Optimal? No

# Depth-limited search

Is DF-search with depth limit l.

- i.e. nodes at depth l have no successors
- Problem knowledge can be used

Solves the infinite-path problem, but

If l < d then incompleteness results

If l > d then not optimal

Time complexity: $O(b^l)$

Space complexity: $O(bl)$

# Depth-limited algorithm

```
function DEPTH-LIMITED-SEARCH(problem,limit) return a solution or failure/cutoff
    return RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]),problem,limit)

function RECURSIVE-DLS(node, problem, limit) return a solution or failure/cutoff
    cutoff_occurred? ← false
    if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
    else if DEPTH[node] == limit then return cutoff
    else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result == cutoff  then cutoff_occurred? ←  true
        else if result ≠ failure then return result
    if cutoff_occurred? then return cutoff else return failure
```

# Iterative deepening search

What?

- A general strategy to find best depth limit *l*
    - Solution is found at depth *d*, the depth of the shallowest solution-node
- Often used in combination with DF-search

Combines benefits of DF- and BF-search

# Iterative deepening search

**function** ITERATIVE_DEEPENING_SEARCH(*problem*)
**return** a solution or failure

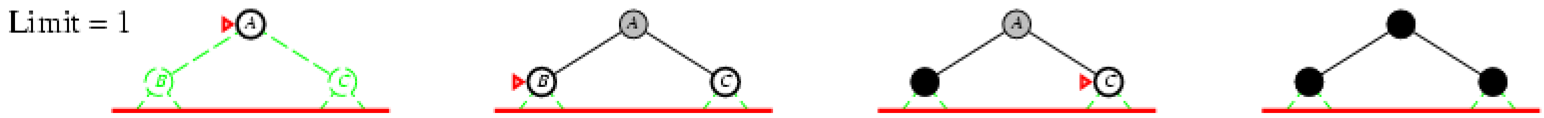
**inputs:** *problem*

**for** *depth* $\leftarrow$ 0 to $\infty$ **do**
*result* $\leftarrow$ DEPTH-LIMITED_SEARCH(*problem*, *depth*)
**if** *result* $\neq$ *cuttoff* **then return** *result*

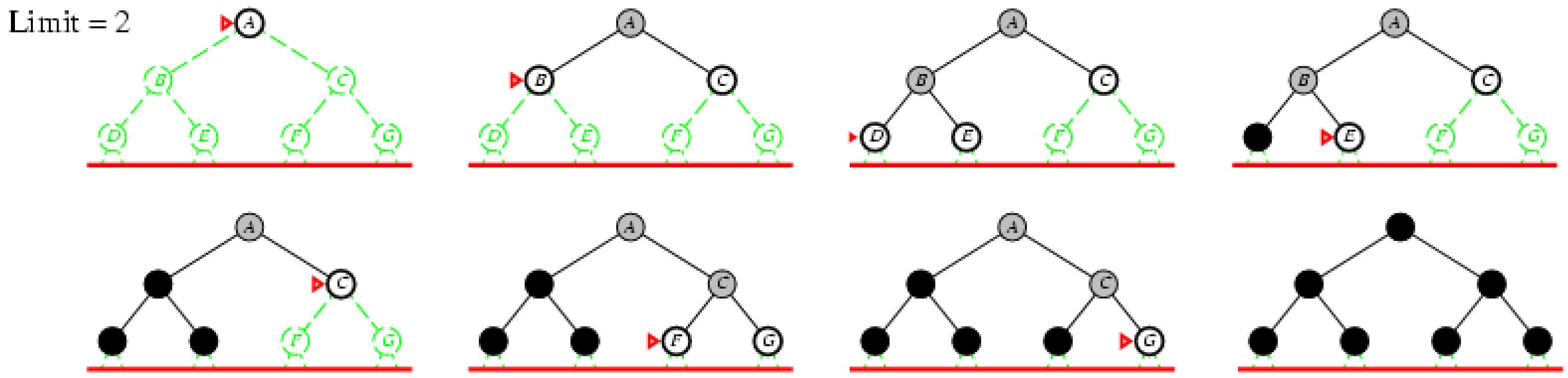
# Iterative deepening search $l$ =0

Limit = 0

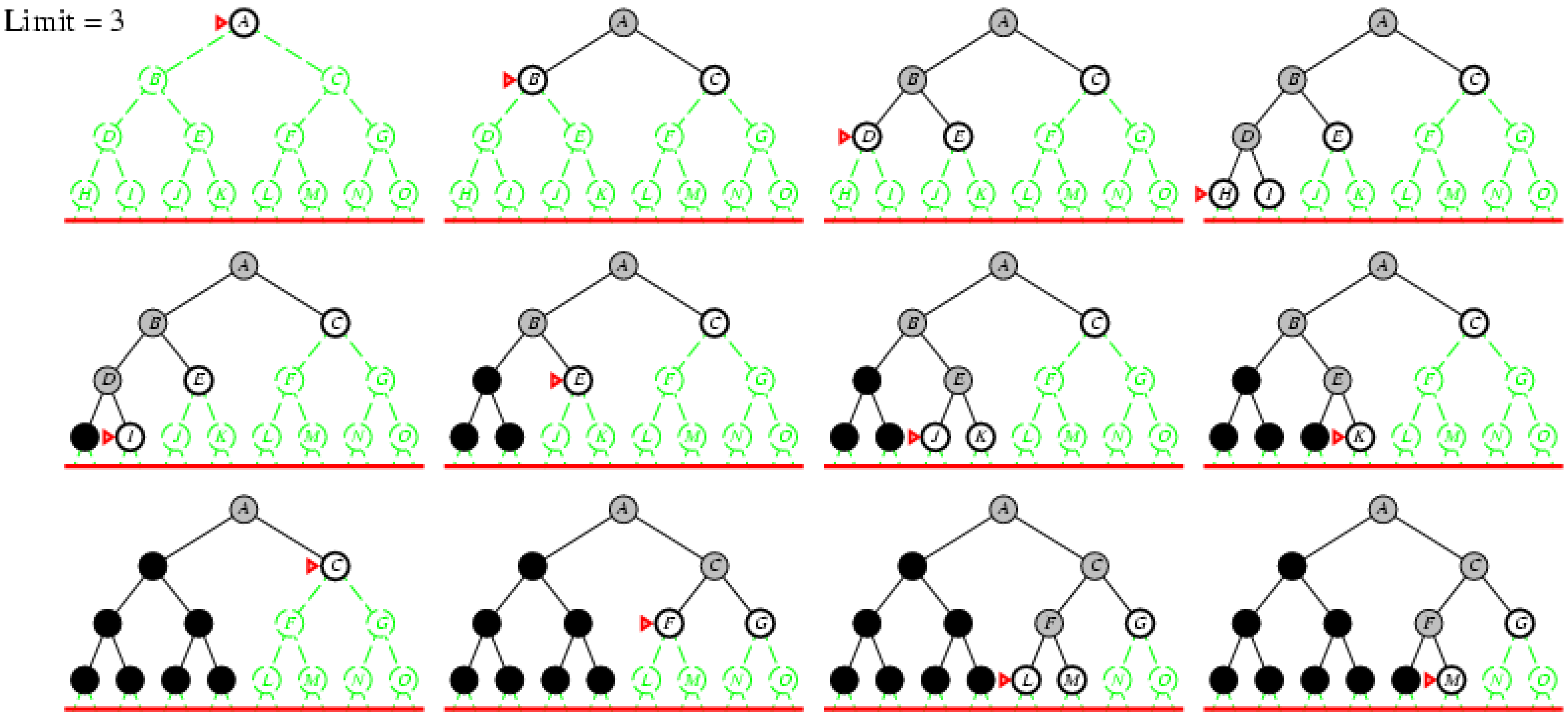
# Iterative deepening search *l* =1



# Iterative deepening search $l$ =2



# Iterative deepening search $l$ =3



# Iterative deepening search

- Number of nodes generated in a depth-limited search to depth *d* with branching factor *b*:

$$N_{DLS} = b^0 + b^1 + b^2 + \ldots + b^{d-2} + b^{d-1} + b^d$$

- Number of nodes generated in an iterative deepening search to depth *d* with branching factor *b*:

$$N_{IDS} = (d+1)b^0 + d\, b^1 + (d-1)b^2 + \ldots + 3b^{d-2} + 2b^{d-1} + 1b^d$$

- For $b = 10$, $d = 5$,
  - $N_{DLS} = 1 + 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 = 111{,}111$
  - $N_{IDS} = 6 + 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}456$

- Overhead = (123,456 - 111,111)/111,111 = 11%

# Properties of iterative deepening search

- Complete? Yes
- Time? $(d+1)b^0 + d\, b^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
- Space? $O(bd)$
- Optimal? Yes, if step cost = 1

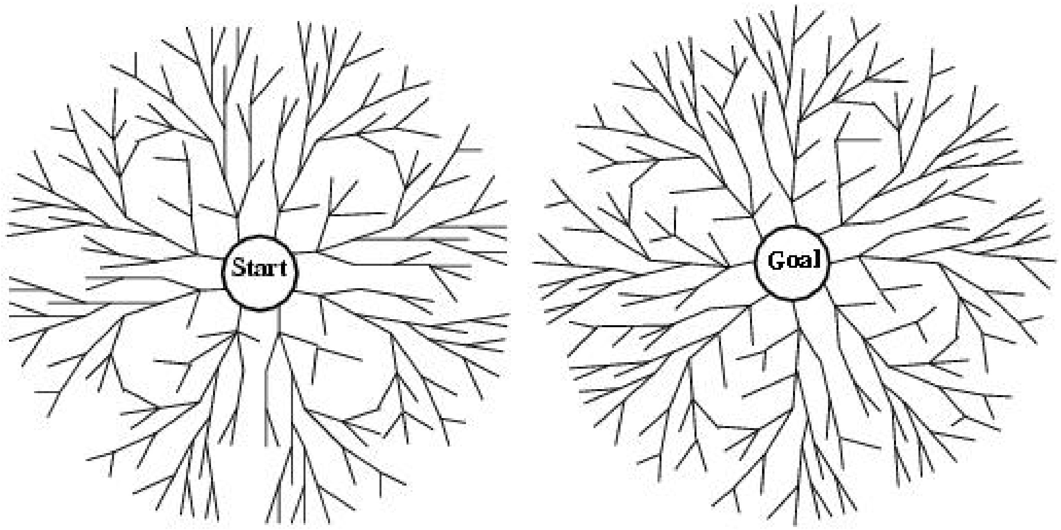
Num. comparison for b=10 and d=5 solution at far right

$N_{IDS} = 6 + 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}456$

$N_{BFS} = 1 + 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 + 999{,}990 = 1{,}111{,}101$

- IDS does better because nodes at depth d are not further expanded
- BFS can be modified to apply goal test when a node is generated

# Bidirectional search




Two simultaneous searches from start an goal

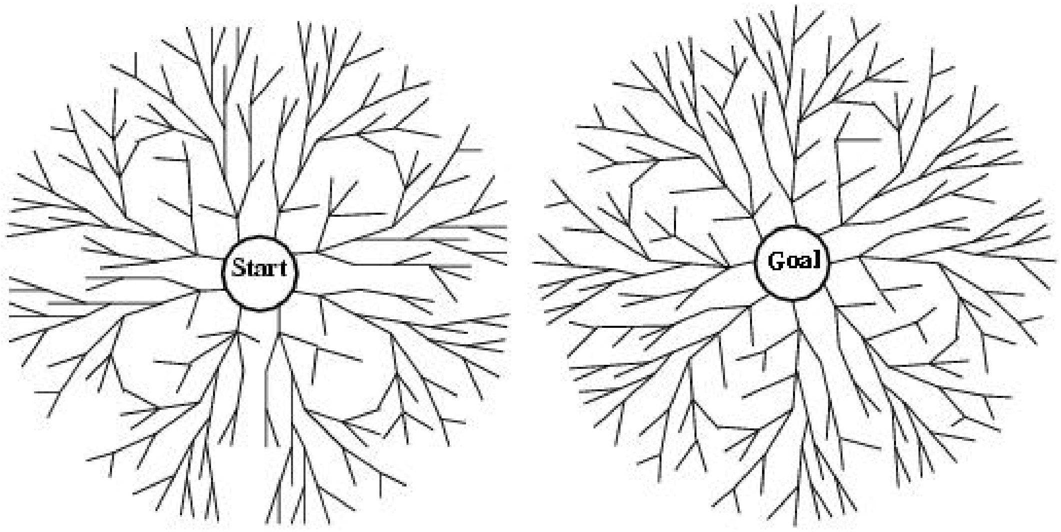
- Motivation: $$b^{d/2} + b^{d/2} \neq b^d$$

Check whether the node belongs to the other fringe before expansion

Complete and optimal if both searches are BF

Space complexity is the most significant weakness

# How to search backwards?




The predecessor of each node should be efficiently computable

- When actions are easily reversible

Number of goal states does not explode

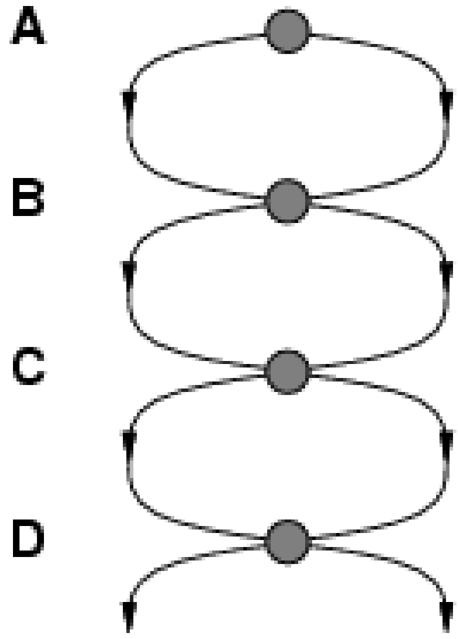
# Summary of algorithms

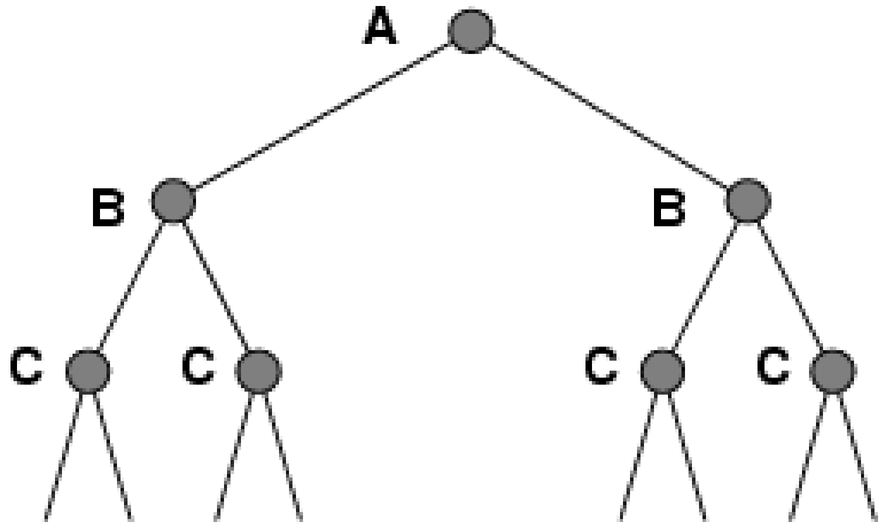
| Criterion | Breadth-First | Uniform-cost | Depth-First | Depth-limited | Iterative deepening | Bidirectional search |
|---|---|---|---|---|---|---|
| **Complete?** | YES[a] | YES[a,b] | NO | YES, if $l \geq d$ | YES[a] | YES[a,d] |
| **Time** | $b^{d+1}$ | $b^{1+floor(C^*/e)}$ | $b^m$ | $b^l$ | $b^d$ | $b^{d/2}$ |
| **Space** | $b^{d+1}$ | $b^{1+floor(C^*/e)}$ | $bm$ | $bl$ | $bd$ | $b^{d/2}$ |
| **Optimal?** | YES[c] | YES | NO | NO | YES[c] | YES[c,d] |

a … if d is finite
b … if step costs >= e
c … if step costs are equal
d … if both directions use BFS

# Repeated states

- Failure to detect repeated states can turn a linear problem into an exponential one!



# Graph search algorithm

"Closed"-list stores all expanded nodes

**function** GRAPH-SEARCH(*problem,fringe*) **return** a solution or failure
    *closed* ← an empty set
    *fringe* ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
    **loop do**
        **if** EMPTY?(*fringe*) **then return** failure
        *node* ← REMOVE-FIRST(*fringe*)
        **if** GOAL-TEST[*problem*] applied to STATE[*node*] succeeds
                **then return** SOLUTION(*node*)
        **if** STATE[*node*] is not in *closed* **then**
                add STATE[*node*] to *closed*
                *fringe* ← INSERT-ALL(EXPAND(*node*, *problem*), *fringe*)

# Graph search, evaluation

Optimality:

- GRAPH-SEARCH discard newly discovered paths
  - This may result in a sub-optimal solution
  - YET: when uniform-cost search or BF-search with constant step cost

Time and space complexity,

- proportional to the size of the state space

  (may be much smaller than $O(b^d)$)
- DF- and ID-search with closed list no longer has linear space requirements since all nodes are stored in closed list!!

# Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms